Cost of Capital

Derivation of Gordon's Formula

V(E) = PV of all Future CIs from the Share

$= PV \text{ of } [D_1 + D_2 + D_3 + D_4 + \ldots \ldots D_\infty]$

Assumption 1: Dividends are Expected to be paid at end of every year.

$= PV \text{ of } D_1 + PV \text{ of } D_2 + PV \text{ of } D_3 + PV \text{ of } D_4 + \ldots\ldots PV \text{ of } D_\infty$

$$= \frac{D_1}{(1+Ke)^1} + \frac{D_2}{(1+Ke)^2} + \frac{D_3}{(1+Ke)^3} + \frac{D_4}{(1+Ke)^4} + \ldots\ldots \frac{D_\infty}{(1+Ke)^\infty}$$

Assumption 2: g in Dividends is Expected to remain constant for ∞.

$D_1 = 100$
$g = 20\%$
$D_2 = 100 + 20\%$ OR $100[1 + 0.2]$
$=$ $D_1[1+g]$

$$V(E) = \frac{D_1}{(1+Ke)^1} + \frac{D_1(1+g)}{(1+Ke)^2} + \frac{D_1(1+g)^2}{(1+Ke)^3} + \frac{D_1(1+g)^3}{(1+Ke)^4} + \ldots\ldots \frac{D_1(1+g)^{\infty-1}}{(1+Ke)^\infty}$$

2 4 6 8 → AP

$\boxed{2 \quad 4 \quad 8 \quad 16}$ → GP

↓

$\Sigma = 30$

$$\Sigma FGP = \frac{a[1-r^n]}{1-r}$$

$$= \frac{2[1-2^4]}{1-2}$$

$$= 30$$

$$\Sigma IGP = \frac{a[1-r^\infty]}{1-r}$$

Condition :

$$0 < r < 1$$

$$\therefore, r^\infty = 0$$

$$\Sigma IGP = \frac{a}{1-r}$$

OR

$$y = a + ar + ar^2 + ar^3 + \ldots\ldots ar^\infty$$

$$y = a + r[a + ar + ar^2 + ar^3 + \ldots\ldots ar^\infty]$$

$$y = a + ry$$

$$y - ry = a$$

$$y[1-r] = a$$

$$y = \frac{a}{1-r}$$

$$V(E) = \frac{D_1}{(1+k_e)^1} + \frac{D_1(1+g)}{(1+k_e)^2} + \frac{D_1(1+g)^2}{(1+k_e)^3} + \frac{D_1(1+g)^3}{(1+k_e)^4} + \ldots\ldots \frac{D_1(1+g)^{\infty-1}}{(1+k_e)^{\infty}}$$

$T② \div T①$

$$\frac{\cancel{D_1}(1+g)}{(1+k_e)^{\cancel{2}}} \times \frac{\cancel{(1+k_e)^1}}{\cancel{D_1}}$$

$$= \frac{1+g}{1+k_e}$$

$T③ \div T②$

$$\frac{\cancel{D_1}(1+g)^{\cancel{2}}}{(1+k_e)^{\cancel{3}}} \times \frac{\cancel{(1+k_e)^2}}{\cancel{D_1(1+g)}}$$

$$= \frac{1+g}{1+k_e}$$

$\because$ common r, this is an IGP series

$0 < \frac{1+g}{1+k_e} < 1 \longrightarrow$ for this, Assumption 3: $k_e > g$

$$V(E) = \Sigma \text{IGP} = \frac{a}{1-r}$$

$$= \frac{\frac{D_1}{1+k_e}}{1-\left[\frac{1+g}{1+k_e}\right]} = \frac{\frac{D_1}{\cancel{1+k_e}}}{\frac{\cancel{1}+k_e-\cancel{1}-g}{\cancel{1+k_e}}} = \frac{D_1}{k_e - g}$$

Assumption 3: $k_e > g$

<u>CAFC</u>:

If $g = 0$

$$V(E) = \frac{D_1}{k_e} \longrightarrow \text{PV of Perpetual Annuity} = \frac{CF}{i}$$

$$\downarrow$$

$$\text{PV of Growing Annuity} = \frac{CF}{i-g}$$

$$P_0 = \frac{D_1}{k_e - g}$$

$$k_e - g = \frac{D_1}{P_0}$$

$$k_e = \frac{D_1}{P_0} + g$$